Midas Intro Session

Buckling Analysis for Slender Structural Members

Midas North American Office
Wednesday, Mar 15th, 2022
3:00 PM – 4:00 PM EDT

Presenter: JC Sun jsun@midasoft.com
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- Concrete Crack Analysis
- Static Analysis
- Fatigue Analysis
- Construction Stage Analysis
- Reinforcement Analysis
- Buckling Analysis
- Eigenvalue Analysis
- Response Spectrum Analysis
- Time History Analysis(Linear/Nonlinear)
- Static Contact Analysis
- Interface Nonlinearity Analysis
- Nonlinear Analysis(Material/Geometric)
- Heat of Hydration Analysis
- Heat Transfer Analysis
- Slope Stability Analysis
- Seepage Analysis
- Consolidation Analysis
- Coupled Analysis(Fully/Semi)

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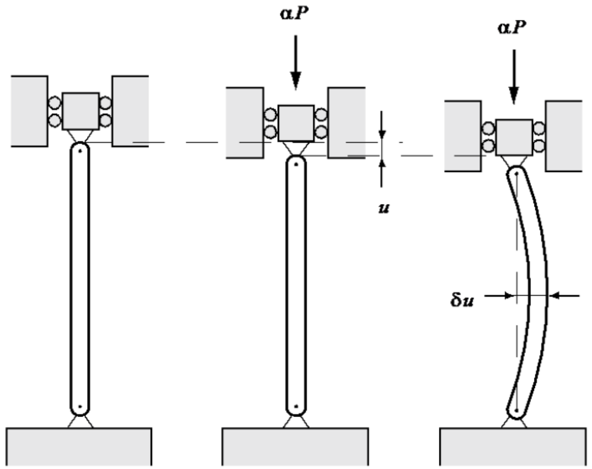
Buckling in Slender Structures and Thin-Walled Members



Buckling is one of the ways load carrying structures may fail. It is the sudden lateral deformation of the slender structure when subjected to excessive axial compressive stress.

If a compression member is relatively slender, it may deflect laterally and fail by bending rather than failing by direct compression of the material. When lateral bending occurs, we say that the structure (e.g. column) has buckled. Under an increasing axial load, the lateral deflections will increase too, and eventually the column will collapse completely.

Buckling is one of the major causes of failures in structures, and therefore the possibility of buckling should always be considered in design. (Gere, Goodno, 2009)

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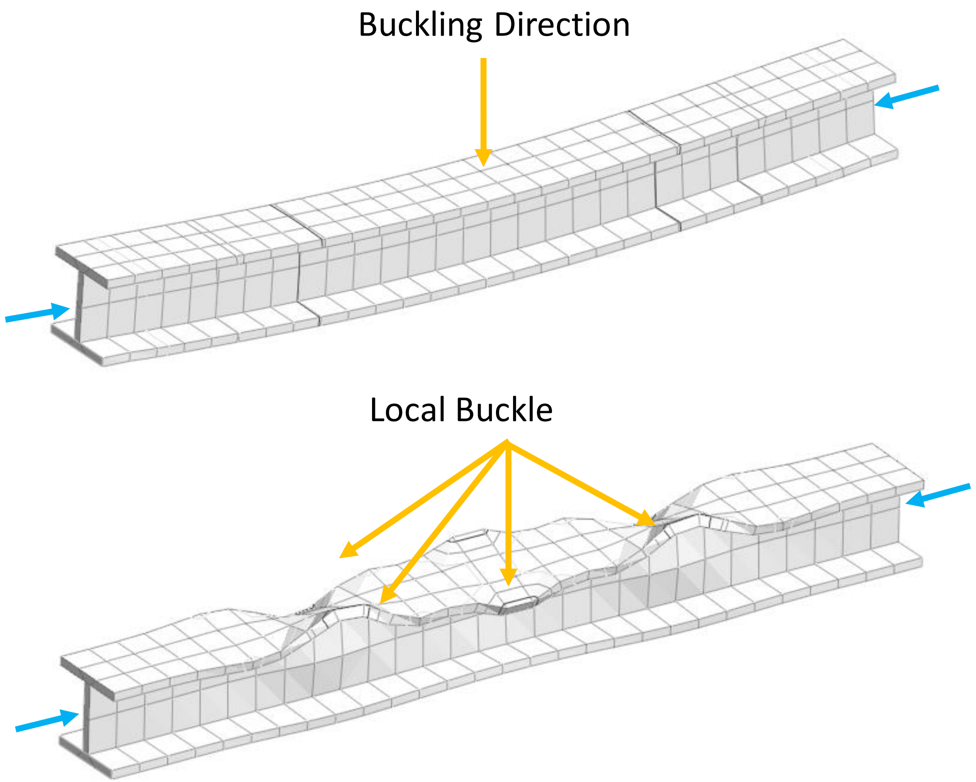

There are usually 3 types of buckling:

- Local
- Distortional
- Global buckling

Interestingly, general and widely adopted definitions for thin-walled buckling modes do not currently exist; however, the following definitions may be considered as the most used:

- Global buckling is a buckling mode where the member deforms with no deformation in its cross-sectional shape, consistent with classical beam theory
- Local buckling is normally defined as the mode which involves plate-like deformation alone, without the translation of the intersection lines of adjacent plate elements
- Distortional buckling is a mode with cross-sectional distortion that involves the translation of some of the fold lines (intersection lines of adjacent plate elements) (Schafer, Adany, 2005)

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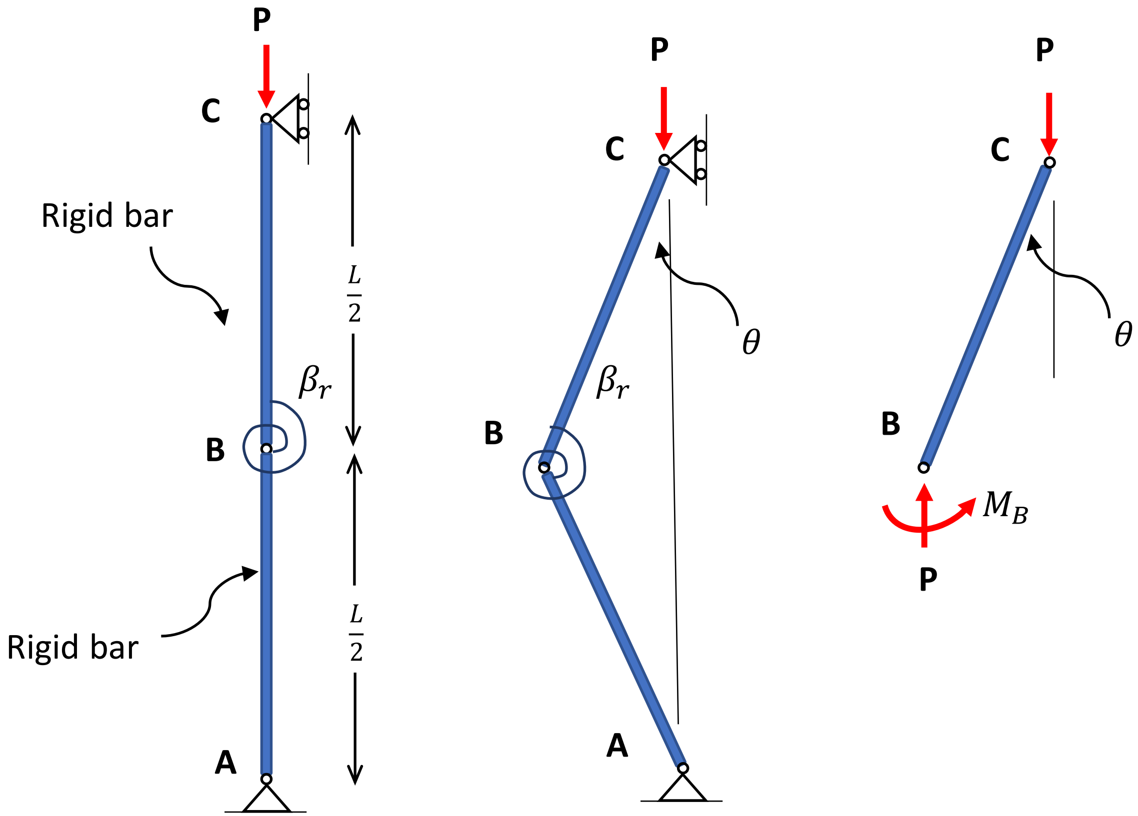
Fundamentals Concepts of Local Buckling

Who's this guy



Buckling Mechanism

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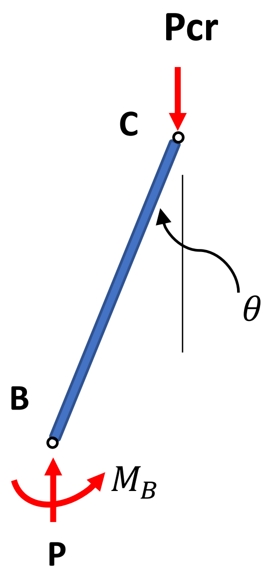

Critical Load

We can determine the critical load of our buckling model by considering the structure in the disturbed position and investigating its equilibrium.

Critical load is the only load for which the structure will be in equilibrium in the disturbed position. At this value of the load, the restoring effect of the moment in the spring just matches the buckling effect of the axial load. Therefore, the critical load represents the boundary between the stable and unstable conditions.

If the axial load is less than Pcr, the effect of the moment in the spring predominates and the structure returns to the vertical position after a slight disturbance; if the axial load is larger than Pcr, the effect of the axial force predominates and the structure buckles:

If P < Pcr, the structure is stable
If P> Pcr, the structure in unstable (Gere, Goodno, 2009)

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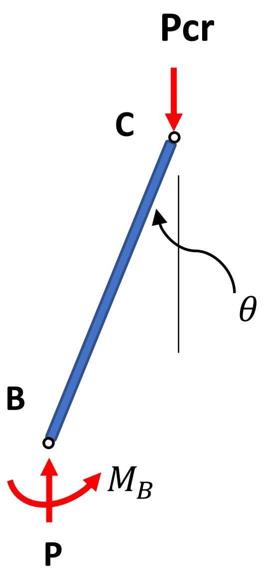
Fundamentals Concepts of Local Buckling – Critical Load



When we consider bar BC as a free body,

$$M_B = 2\beta_r \theta$$

Since the angle θ is a small quantity, the lateral displacement of point B is $\frac{\theta L}{2}$. By writing down the moment equilibrium at point B,

$$M_B - P\left(\frac{\theta L}{2}\right) = 0$$

When substituting in the first equation, we get

$$\left(2\beta_r - \frac{PL}{2}\right) = 0 \quad \rightarrow \quad P_{cr} = \frac{4\beta_r}{L}$$

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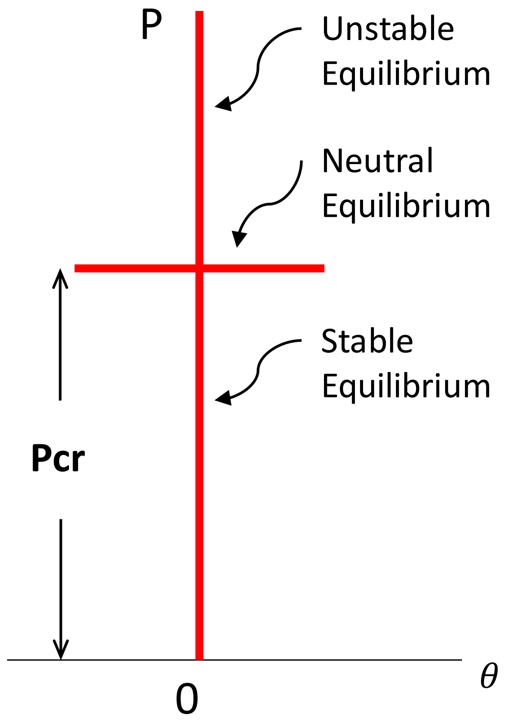
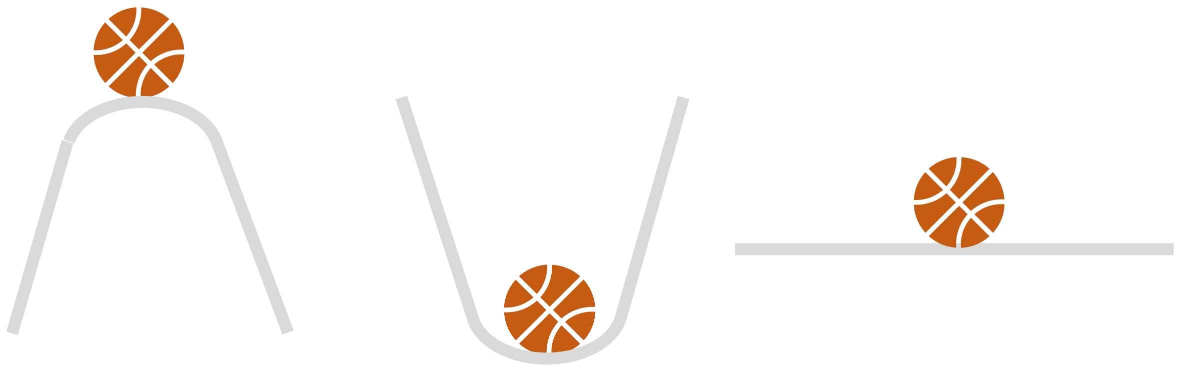
Fundamentals Concepts of Local Buckling – Critical Load



When the axial load is larger than the critical load (P > Pcr), the equilibrium is **unstable**, meaning $\theta = 0$ when there's no moment applied. However, a slightest disturbance will drive the system towards failure.

When the axial load is equal to the critical load (P = Pcr), the condition is **neutral equilibrium**, and it is neither stable nor unstable. For a small disturbance θ the structure would not go back to the original position but would also not go to complete failure.

When the axial load is less than the critical load (P < Pcr), the equilibrium is **stable**, and the structure will return to its original position after a small disturbance is applied.



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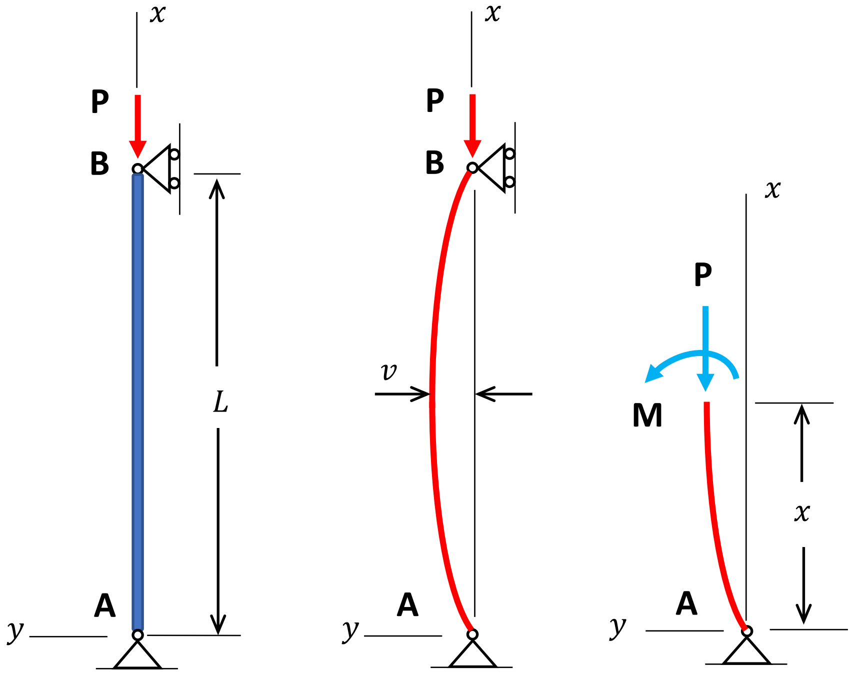
Fundamentals Concepts of Local Buckling – Critical Load



The bending – moment equation is

$$EIv'' = M$$

At point A, the equilibrium of moments is

$$M + Pv = 0, \qquad \text{or } M = -Pv$$

Therefore, the first equation becomes

$$EIv'' + Pv = 0,$$

$$v'' + \frac{P}{EI}v = 0,$$

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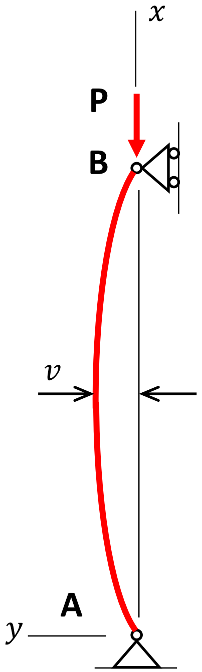
Fundamentals Concepts of Local Buckling – Critical Load



$$v'' + \frac{P}{EI}v = 0 \ ???$$

Out of convenience, we use a quantity k

$$k^2 = \frac{P}{EI} \text{ or } k = \sqrt{\frac{P}{EI}}$$

$$\therefore v'' + k^2 v = 0$$

We know the general solution to the equation above is

$$v = C1 \sin kx + C2 \cos kx$$

From boundary conditions,

$$v(0) = 0 \ \ and \ \ v(L) = 0$$

Plugging in the previous equation, *v(0)=0* indicates that *C2 =0*

The second condition gives

$$C1 \sin kL = 0$$

Therefore, $\sin kL = 0$, to satisfy this kL = 0, $\pi . 2\pi$, ….

$$kL = n\pi \qquad n = 1,2,3, \ldots.$$

$$\boxed{\therefore P = \frac{n^2\pi^2 EI}{L^2},} \qquad v = C1 \sin kx = C1 \sin \frac{n\pi x}{L}$$

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Fundamentals Concepts of Local Buckling – Critical Load

When n = 1, we obtain the lowest critical load,

$$P_{cr} = \frac{n^2\pi^2 EI}{L^2}$$

Buckled shapes for the higher nodes are often of no practical interest because the column buckles when the axial load P reaches its lowest critical value. The only way to obtain modes of buckling higher than the first is to provide lateral support of the column at intermediate points, such as at the midpoint of the column. (Gere, Goodno, 2009)

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Fundamentals Concepts of Local Buckling – Critical Stress and Euler's Curve

Critical stress can be determined as follows

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 EI}{AL^2}$$

When written in a more useful form,

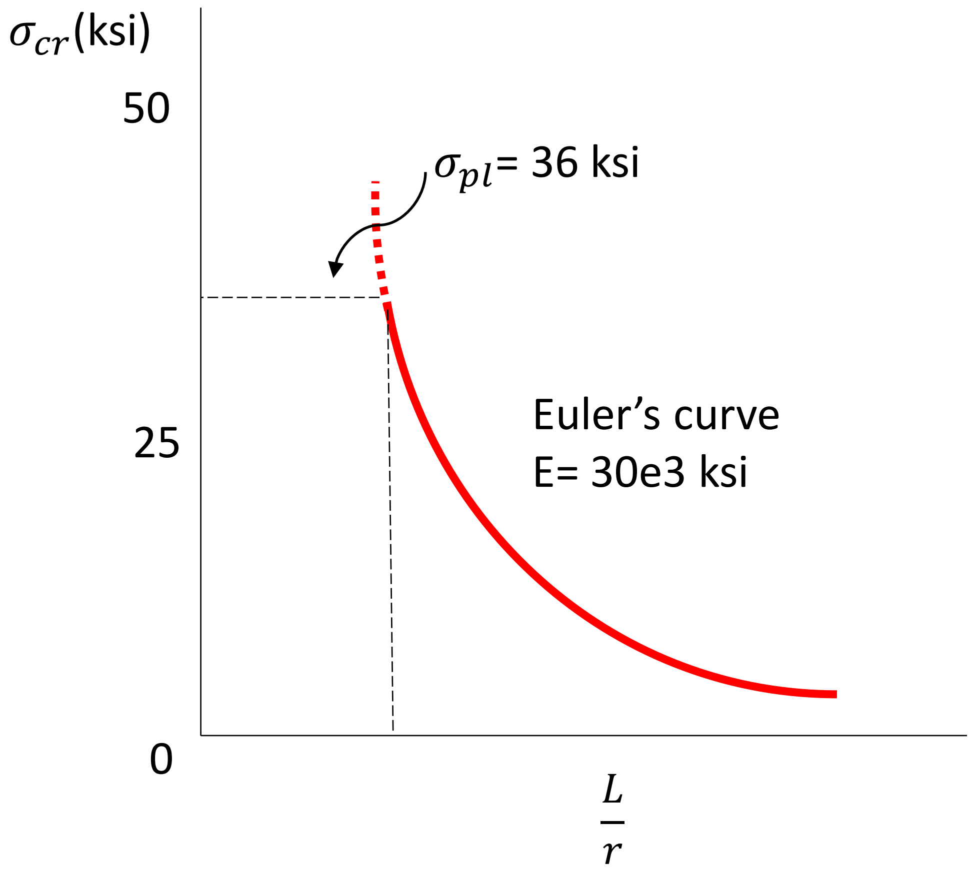
$$r = \sqrt{\frac{I}{A}}$$

, where r is the radius of gyration, and

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 E}{(L/r)^2}$$

, where r is the radius of gyration, and L/r is the slenderness ratio

Slenderness ratio = $\frac{L}{r}$



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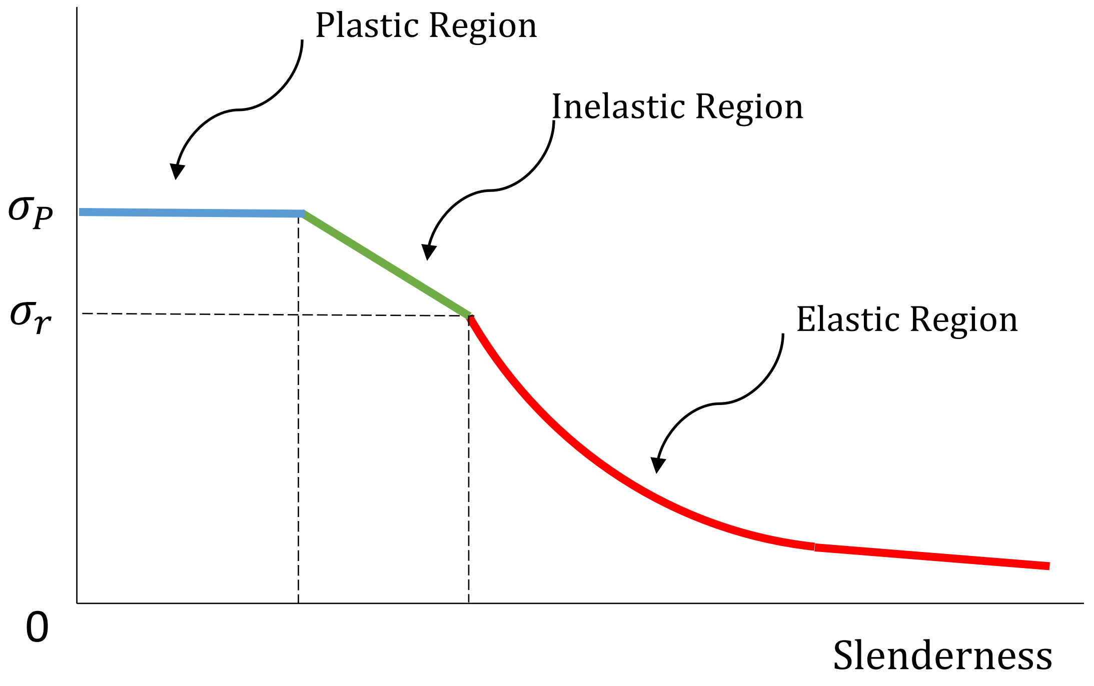
Fundamentals Concepts of Local Buckling – Critical Stress and Euler's Curve



In the case of global buckling, the ranges are known as the plastic, inelastic, and elastic ranges. When the member slenderness is low, then plastic behavior is expected, and the member strength is limited by plastic strength of the material. When the member slenderness is in the transition range, then inelastic behavior and member strength is a combination of plastic and elastic strength. Finally, when member slenderness is large then elastic buckling behavior is expected and the strength of the member is controlled by Euler buckling behavior.

For the case of local buckling, the slenderness is based on width/thickness ratios of the slender plate elements that make up the cross section of most steel members. The member cross sections are then classified by which of the 3 ranges their most slender element falls in. (Quimby, 2008)

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Reference

Gere, James M., Goodno, Barry J. *Mechanics of Materials, 7th edition,* Cengage Learning, 2009

Schafer, B.W., Adany, S. *Buckling Analysis of Cold-Formed Steel Members Using CUFSM Conventional and Constrained Finite Strip Methods*, 18th International Specialty Conference on Cold-Formed Steel Structures, 2005

Quimby, Bartlett *A Beginner's Guide to the Steel Construction Manual, 13th ed*, 2008

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